Discretized Light-Cone Quantization of Two-Dimensional SU(N) Gauge Theories

Senior Thesis

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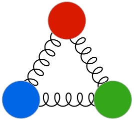
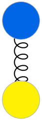
Motivation: color confinement

Quarks never isolated in the real world – always confined to color-neutral multi-particle states:

e.g. baryons (qqq)  and mesons ($q\bar{q}$) 

Try describing the force between quarks/antiquarks via an interaction potential $V(r)$, similar to Coulomb potential ($\sim r^{-1}$) for electron/positron interactions in E&M.

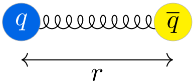
The Cornell potential

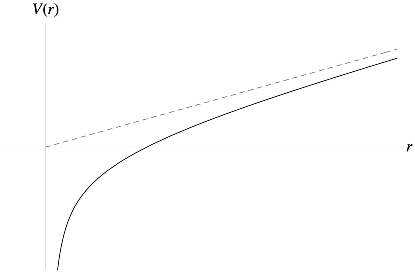
Numerical simulations and experiments [1, 2] show quark-antiquark potential grows linearly at large separations, e.g.

$$V(r) = -\frac{\kappa}{r} + \beta r \tag{1}$$

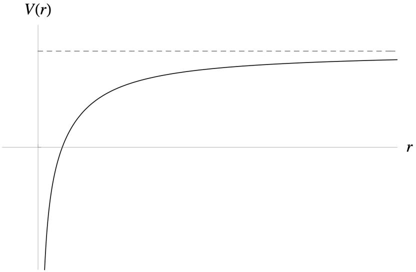
In the real world, figure out κ, β by measuring the masses of mesons.

Confining vs. screening





(a) Confining, $V(r \to \infty) \sim r$.



(b) Screening, $V(r \to \infty) \sim \text{const.}$

Figure: Schematic plots of confining and screening quark-antiquark potentials. Viewed as quantum mechanical potentials, (a) can only have bound states corresponding to mesons, whereas (b) must have scattering states corresponding to screened quarks (and may also have bound states).

This thesis (vs. real QCD)

Real-world QCD:

- Quarks with 3 types of color charge
- Gluons with 8 types of color charge

Understanding confinement analytically in real-world QCD is still an open problem. Need simpler models.

This thesis: 2d adjoint QCD.

- $(1+1)$-d instead of $(3+1)$-d
  - Gluon becomes non-dynamical
  - $V(r \to 0) \sim r$ instead of $V(r \to 0) \sim -r^{-1}$
- N quark colors instead of 3 ($\implies$ $N^2 - 1$ gluon colors)
- **Gluinos** with $N^2 - 1$ types of color charge
- Arbitrary number N_{adj} of gluino *flavors*

How to study this theory?

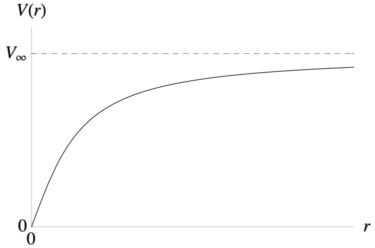
Compute masses of "observable" particles (i.e. energy eigenstates). Analogous to finding masses of baryons & mesons in the real world.

This thesis: studying mass spectrum of 2d adjoint QCD

At $N = 2$, spectrum not studied much due to numerical difficulties [3].

Step 1: Develop techniques for the pure gluino theory at $N = 2$.

Step 2: Add probe quarks so we can study confining/screening behavior.



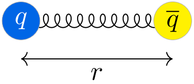


Figure: Form of quark-antiquark potential in large N theory.

At large N, much easier to get the spectrum. Expect screening when gluinos are massless [4].

$\implies$ Can extract **height of quark-antiquark potential**, V_∞.
(In fact, we showed numeric result agrees with analytic calculation.)

N is for numerics

Start with action

$$S = \int \mathrm{d}^2x \Big[\mathrm{Tr}\Big(\frac{-1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_{\alpha=1}^{N_{\text{adj}}} \overline{\Psi}_\alpha \big(i\not{D} - m_{\text{adj}} \big) \Psi_\alpha \Big) + \overline{q} \big(i\not{D} - m_{\text{fund}} \big) q \Big] \quad (2)$$

QCD very strongly coupled, so can't use perturbation theory. What to do?

Numerical method: discretized light-cone quantization (DLCQ) [5]

1. Switch to light-cone coordinates $x^\pm = (t \pm x)/\sqrt{2}$
2. Compactify x^- to a circle of radius L: $x^- \sim x^- + 2\pi L$
3. Compute operators $P^\pm = (E \pm P)/\sqrt{2} \implies M^2 = E^2 - P^2 = 2P^+P^-$
4. Act with creation operators $B^\dagger(n)$ on the vacuum to make states with momentum $P^+ = K/(2L)$
5. "Continuum limit" defined by $K \to \infty$, $L \to \infty$ while P^+ fixed

2 types of states [6]: gluinoballs & mesons.

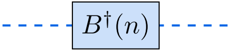
New techniques for gluinoballs at $N = 2$

Naively we get lots more states at finite N. Many of them are linearly dependent, but $1/N$ corrections make inner products hard to compute.

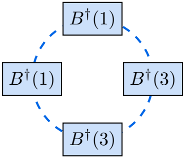
E. g. at momentum $K = 30$ there are 308,226 states; only 158 independent.

Problem 1: How to efficiently enumerate a basis?

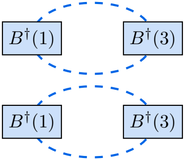
First, think about how to represent creation operators. At large N we use



Make a state by connecting these:



or



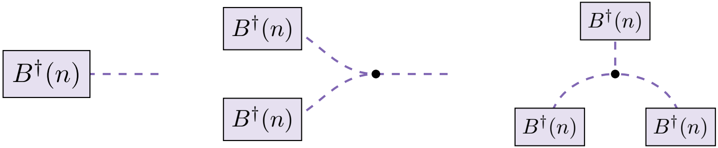
$(K = 8)$

Enumerating all possible ways to do this (on all possible n's) gets us the basis.

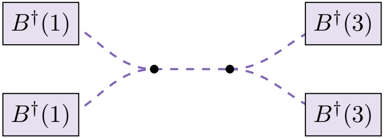
New techniques for gluinoballs at $N = 2$

We *could* use the same representation for $N = 2$, but we'd get null states.

Better to use the following "building blocks"



Example state:



$(K = 8)$

Enumerating states this way gets us a basis very efficiently.

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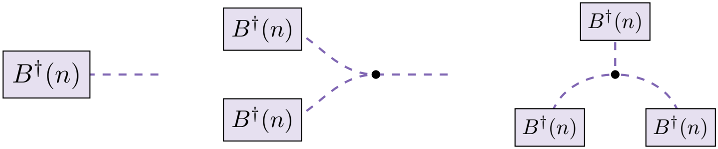
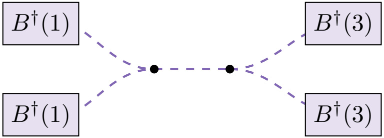


Example state:



$(K = 8)$

Enumerating states this way gets us a basis very efficiently.

Problem 2: How to compute P^- in this representation? (Figured this out too, but won't explain here.)

Results for gluinoballs at $N = 2$

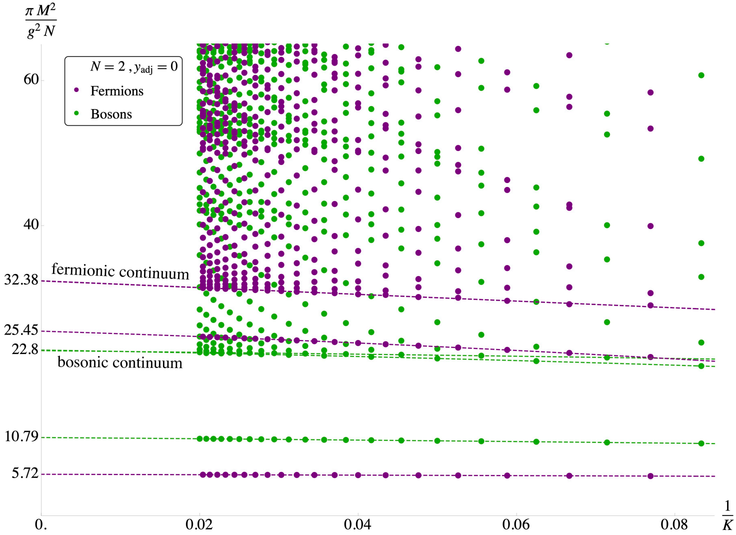


Figure: M^2 eigenvalues of gluinoball states when $m_{\text{adj}} = 0$.

Quark-antiquark potential at large N & large N_{adj}

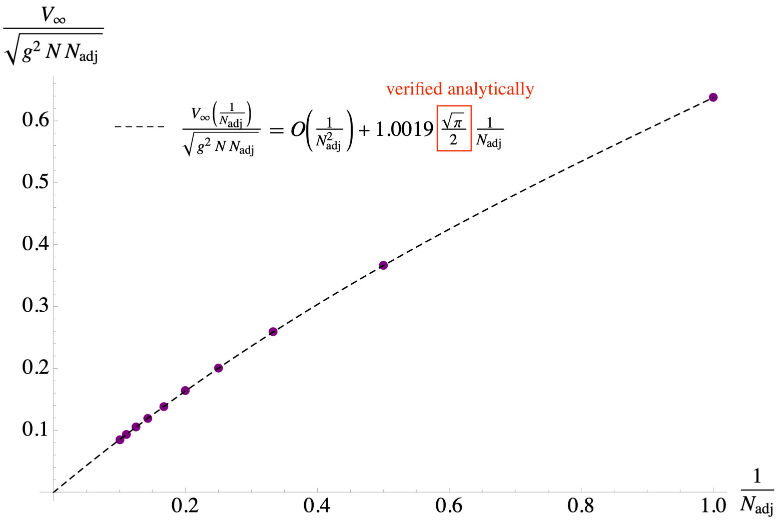


Figure: V_∞ when gluinos are massless, plotted as a function of $1/N_{\text{adj}}$.

Takeaways & next steps

Summary:

- At $N = 2$, developed new techniques using SU(2) representation theory to make DLCQ treatment possible
- At large N, extracted V_∞ from numerics and showed agreement with analytic calculation
- Found unexpected similarities and differences between large and finite N

Suggestions for future research:

- Extend $N = 2$ methods to theory with probe quarks so that SU(2) quark-antiquark potential can be computed numerically
- Extend $N = 2$ methods to $N = 3$
- ...and others

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